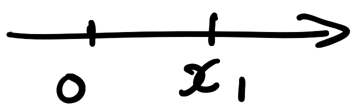
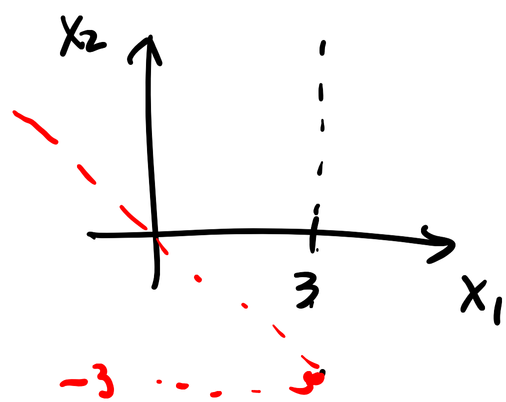
Lec 1

Systems of linear equations.

$$x_1 = 1$$

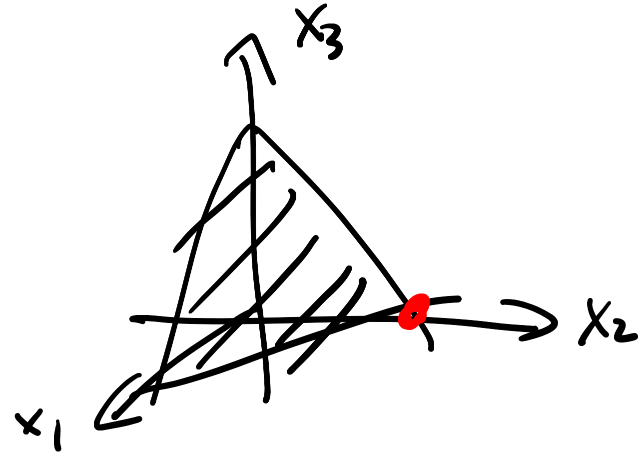


$$\begin{cases} x_1 + x_2 = 0 \\ x_1 = 3 \end{cases}$$



$$x_2 = -x_1 = -3$$

$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 = 0 \\ x_2 = 1 \end{cases}$$



$$x_3 = 1 - x_1 - x_2 = 0$$

Def A linear equation in n variables is an eqn. in the form

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b$$

A system of linear eqns / linear systems in n variables is a finite collection of linear eqns.

$$\begin{cases} a_{11}x_1 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + \cdots + a_{2n}x_n = b_2 \\ \quad\vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n = b_m \end{cases}$$

<u>Def</u> A solution set of a lin. sys. is the set of tuples $(s_1, \cdots, s_n)$ that solve all eqns. in the lin. sys.

Ex. Find sol. set for.

1) $\begin{cases} 3x_1 - x_2 = 0 \\ 2x_1 = 6 \end{cases}$ $\Rightarrow$ sol set $\{(3,9)\}$.

$(3,9) \in \{(3,9)\}$.

$(3,9) = (3,9)$

2) $\begin{cases} 3x_1 + x_2 = 1 \\ -6x_1 - 2x_2 = 0 \end{cases}$ $\Rightarrow$ sol set is ϕ

$\Updownarrow$

$3x_1 + x_2 = 0$

$\downarrow$

empty set.

3) $\begin{cases} x_1 - x_2 + x_3 = 0 \\ x_2 + x_3 = 0 \end{cases}$ $\Rightarrow$ sol set $\{(-2s, -s, s) \mid s \in \mathbb{R}\}$

$\mathbb{R}$: set of all real numbers.

$\mathbb{C}$: " " " complex ".

Only three possible outcomes

1) no sol $\rightarrow$ inconsistent

2) unique sol.

3) infinitely many sols

$\left.\begin{array}{l} \text{2) unique sol.} \\ \text{3) infinitely many sols} \end{array}\right\}$ consistent

<u>Def</u> Two linear sys. are equivalent if they have the same sol set.

Two sets A, B. $A \overset{?}{=} B$

verify

1) $\forall x \in A \Rightarrow x \in B$

2) $\forall y \in B \Rightarrow y \in A$

why $\{(-2s, -s, s) \mid s \in \mathbb{R}\} \leftarrow A$

$= \{(4s, 2s, -2s \mid s \in \mathbb{R}\} \leftarrow B$.

1) $\forall\ (-2s, -s, s) \in A$, multiply (-2)

$\rightarrow (4s, 2s, -2s) \in B$

2) exer.

Ex. For which value of c are the following lin. systems equivalent?

① $\begin{cases} x_1 - c x_2 = 0 \\ x_1 + x_3 = 0 \end{cases}$ ② $\begin{cases} 2x_1 - x_2 + x_3 = 0 \\ x_2 + x_3 = 0 \end{cases}$.

step 1. Find each sol set

① $\{(cs, s, -cs) \mid s \in \mathbb{R}\} \leftarrow A$

② $\{(-t, -t, t) \mid t \in \mathbb{R}\} \leftarrow B$

Step 2. Find c

Pick $(-1,-1,1) \in B$

$$\Rightarrow \begin{cases} cs = -1 \\ s = -1 \\ -cs = 1 \end{cases} \Rightarrow c = 1$$

Step 3. Verify equivalence

$$\{(s,s,-s) \mid s \in \mathbb{R}\} = \{(-t,-t,t) \mid t \in \mathbb{R}\}$$