Lec 13

<u>Ex</u>. $\mathbb{P}_2$ basis $\mathcal{B} = \{1-2t+t^2, 3-5t+4t^2, 2t+3t^2\}$

$C = \{1, 2t, t^2\}$. $[\vec{v}]_C = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

(1) Compute $\underset{C \leftarrow B}{P}$

(2) Use $\underset{C \leftarrow B}{P}$ to compute $[\vec{v}]_{\mathcal{B}}$.

(1) $\underset{C \leftarrow B}{P} = \left[[\vec{b}_1]_C \; , \; [\vec{b}_2]_C \; , \; [\vec{b}_3]_C \right]$

$$= \begin{bmatrix} 1 & 3 & 0 \\ -1 & -\frac{5}{2} & 1 \\ 1 & 4 & 3 \end{bmatrix}$$

(2) $[\vec{v}]_C = \underset{C \leftarrow B}{P} \, [\vec{v}]_B \, .$

$\left(\vec{b} = A \vec{x} \right)$

$$\Rightarrow [\vec{v}]_B = \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$$

Ex. $T : \mathbb{P}_3 \to \mathbb{P}_2$.

$$f(x) \mapsto x \frac{d^2 f}{dx^2}(x)$$

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3.$$

$$[T(f)](x) = x \cdot (2a_2 + 6a_3 x)$$

$$= 2a_2 x + 6a_3 x^2 \in \mathbb{P}_2$$

Find matrix rep. of T w.r.t.

basis $\mathcal{B} = \{1, x, x^2, x^3\}$, $C = \{1, x, x^2\}$

$$A = \begin{bmatrix} [T(\vec{b}_1)]_C & \cdots & [T(\vec{b}_4)]_C \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \boxed{2} & 0 \\ 0 & 0 & 0 & \boxed{6} \end{bmatrix} \in \mathbb{R}^{3 \times 4}.$$

free pivots

$$\dim \mathrm{Null}(T) = \dim \mathrm{Null}(A) = 2$$

$$\dim \mathrm{Image}(T) = \dim \mathrm{Image}(A) = 2$$

$$\begin{array}{c} ||| \\ \mathrm{Rank}(T) \end{array}$$

(Rank theorem) $T: V \to W$

$$\dim V = \mathrm{rank}(T) + \dim \mathrm{Null}(T)$$

$\# \text{col} = \# \text{pivots} + \# \text{free}$.

Relation between

change of coordinates

and

matrix rep. of lin trans.?

Identity map. $I(\vec{x}) = \vec{x}$

$$\begin{tikzcd} I: & V \arrow[r] & V \arrow[d, "P_C^{-1}"] & \\ \mathcal{B} & \mathbb{R}^n \arrow[u, "P_B"] \arrow[r] & \mathbb{R}^n & \mathcal{C} \end{tikzcd}$$

mat. rep.

$$A = P_C^{-1} P_B \equiv \underset{C \leftarrow B}{P}$$