Lec 3

Linear combination of vectors.

$\vec{x}, \vec{y} \in \mathbb{R}^n$, $\quad a, b \in \mathbb{R}$

$$a\vec{x} + b\vec{y} = \begin{bmatrix} ax_1 + by_1 \\ ax_2 + by_2 \\ \vdots \\ ax_n + by_n \end{bmatrix}$$

Special cases

1) addition of 2 vectors.

$$\vec{x} + \vec{y} = \begin{bmatrix} x_1 + y_1 \\ \vdots \\ x_n + y_n \end{bmatrix}$$

2) scalar multiplication

$$a\vec{x} = \begin{bmatrix} ax_1 \\ \vdots \\ ax_n \end{bmatrix}$$

$$a_1\vec{x}_1 + \cdots + a_m\vec{x}_m = \begin{bmatrix} a_1(\vec{x}_1)_1 + \cdots + a_m(\vec{x}_m)_1 \\ \vdots \\ a_1(\vec{x}_1)_n + \cdots + a_m(\vec{x}_m)_n \end{bmatrix}$$

zero vector $\vec{0} = \mathbf{0} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$

$$-\vec{x} = \begin{bmatrix} -x_1 \\ \vdots \\ -x_n \end{bmatrix}$$

Caution :

1) Cannot add vectors of diff. sizes.

2) cannot multiply 2 vectors. (yet)

$$\left[\begin{array}{ccc|c} a_{11} & \cdots & a_{1n} & b_1 \\ \vdots & & \vdots & \vdots \\ a_{m1} & \cdots & a_{mn} & b_m \end{array}\right]$$

$\underbrace{\phantom{a_{11} \cdots a_{1n}}}_{A}$ $\vec{b}$

matrix

$A = [\vec{a}_1 \cdots \vec{a}_n]$

vector eq.

$$x_1 \vec{a}_1 + \cdots + x_n \vec{a}_n = \vec{b}$$

$$|||$$

$$A\vec{x} = \vec{b} \qquad \vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

Homogeneous lin. sys.

$$A = [\vec{a}_1 \cdots \vec{a}_n] \in \mathbb{R}^{m \times n}$$

$$x_1 \vec{a}_1 + \cdots + x_n \vec{a}_n = \vec{0}$$ → of size m

1) $\vec{x} = \vec{0}$ is always a solution.

of size n

called trivial ".

any non-zero sol is called non-trivial sol.

2) If $\vec{x}$ is a non-trivial sol.

then $c\vec{x}$ is also a non-trivial sol.

$\forall c \in \mathbb{R}$, $c \neq 0$.

Sol set Either unique or inf. many.

$\Downarrow$

$\vec{0}$ is the only sol.

$A\vec{x} = \vec{0}$

Ex. $A = \begin{bmatrix} 2 & -5 & 8 \\ -2 & -4 & 1 \\ 4 & -1 & 7 \end{bmatrix}$

Augmented matrix

$$\left[\begin{array}{ccc|c} 2 & -5 & 8 & 0 \\ -2 & -4 & 1 & 0 \\ 4 & -1 & 7 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 2 & -5 & 8 & 0 \\ 0 & -9 & 9 & 0 \\ 0 & 9 & -9 & 0 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 2 & -5 & 8 & 0 \\ 0 & -9 & 9 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$ REF ✓

$$\to \left[\begin{array}{ccc|c} 1 & -\frac{5}{2} & 4 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$ RREF ✗

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & \frac{3}{2} & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

RREF ✓

$$\begin{cases} x_1 + \frac{3}{2}x_3 = 0 \\ x_2 - x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = -\frac{3}{2}x_3 \\ x_2 = x_3 \end{cases}$$

x_3 is a free variable.

$\downarrow$

column has no pivot

$$\text{Sol set } \left\{ \left(-\frac{3}{2}x_3, x_3, x_3\right) \middle| x_3 \in \mathbb{R} \right\}$$

$$= \left\{ x_3 \begin{bmatrix} -\frac{3}{2} \\ 1 \\ 1 \end{bmatrix} \middle| x_3 \in \mathbb{R} \right\}$$

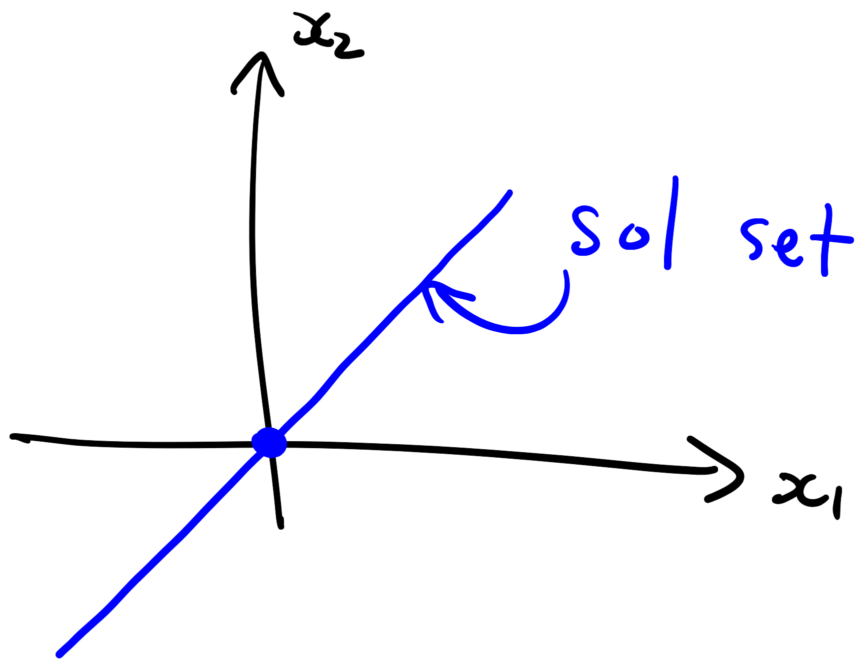
Geometric perspective.

$a_{11} \neq 0.$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \xrightarrow{REF} \left[\begin{array}{cc|c} \boxed{a_{11}} & a_{12} & 0 \\ 0 & 0 & 0 \end{array}\right]$$

if inf many sol.

$$\begin{cases} x_1 = -\dfrac{a_{12}}{a_{11}} x_2. \\ x_2 \text{ is free.} \end{cases}$$



$a_{11} = 1, \quad a_{12} = -1$