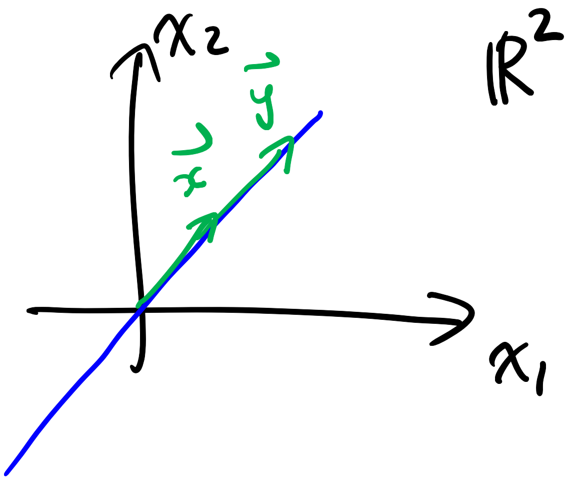
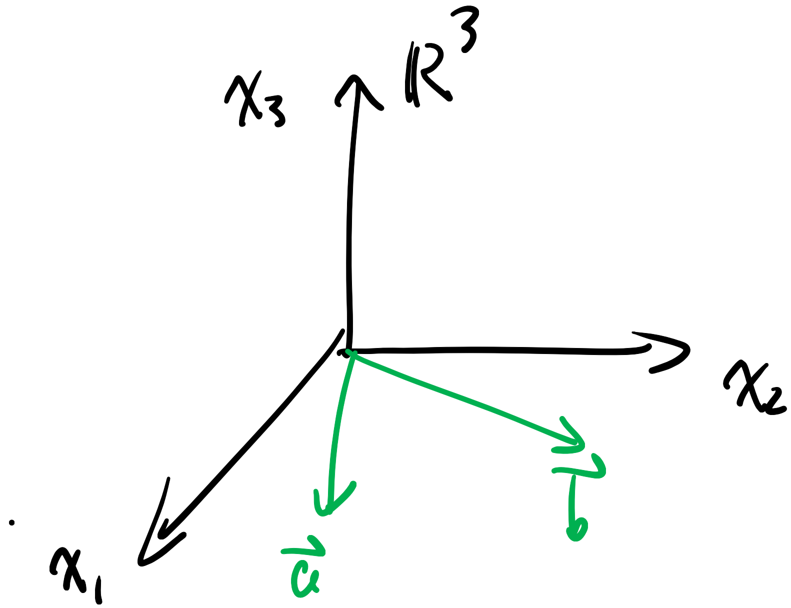
Lec 8 subspace.





Def A subspace H of $\mathbb{R}^n$ is a subset of vectors in $\mathbb{R}^n$ s.t.

(1) $\vec{0} \in H$ H cannot be empty

(2) $\vec{u}, \vec{v} \in H$, then $\vec{u}+\vec{v} \in H$ closed under addition

(3) $\vec{u} \in H$, $c \in \mathbb{R}$ then $c\vec{u} \in H$ closed under scalar mult.

Ex. $\mathbb{R}$. Possible subspaces.

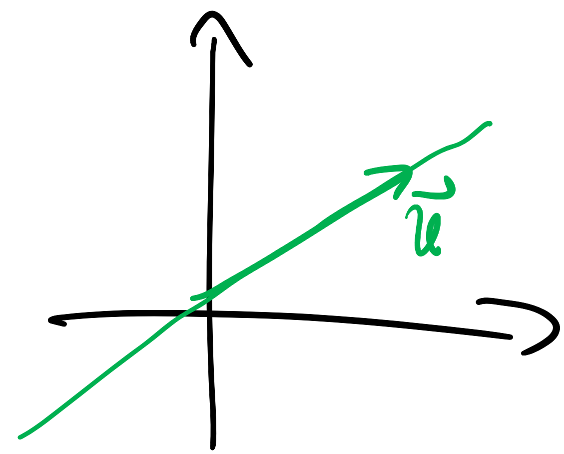
$\{0\}$ $\{0, 1, 2\}$ ×

$\mathbb{R}$ 2 subspaces.

Ex. $\mathbb{R}^2$ possible subspaces.

$\{\vec{0}\}$. $\mathbb{R}^2$.

$\text{span}\{\vec{u}\}$, $\vec{u} \in \mathbb{R}^2, \vec{u} \neq \vec{0}$



Ex. Sol set to hom. lin. sys.

$A\vec{x} = \vec{0}$ $\quad S$. $\quad A \in \mathbb{R}^{m\times n}$

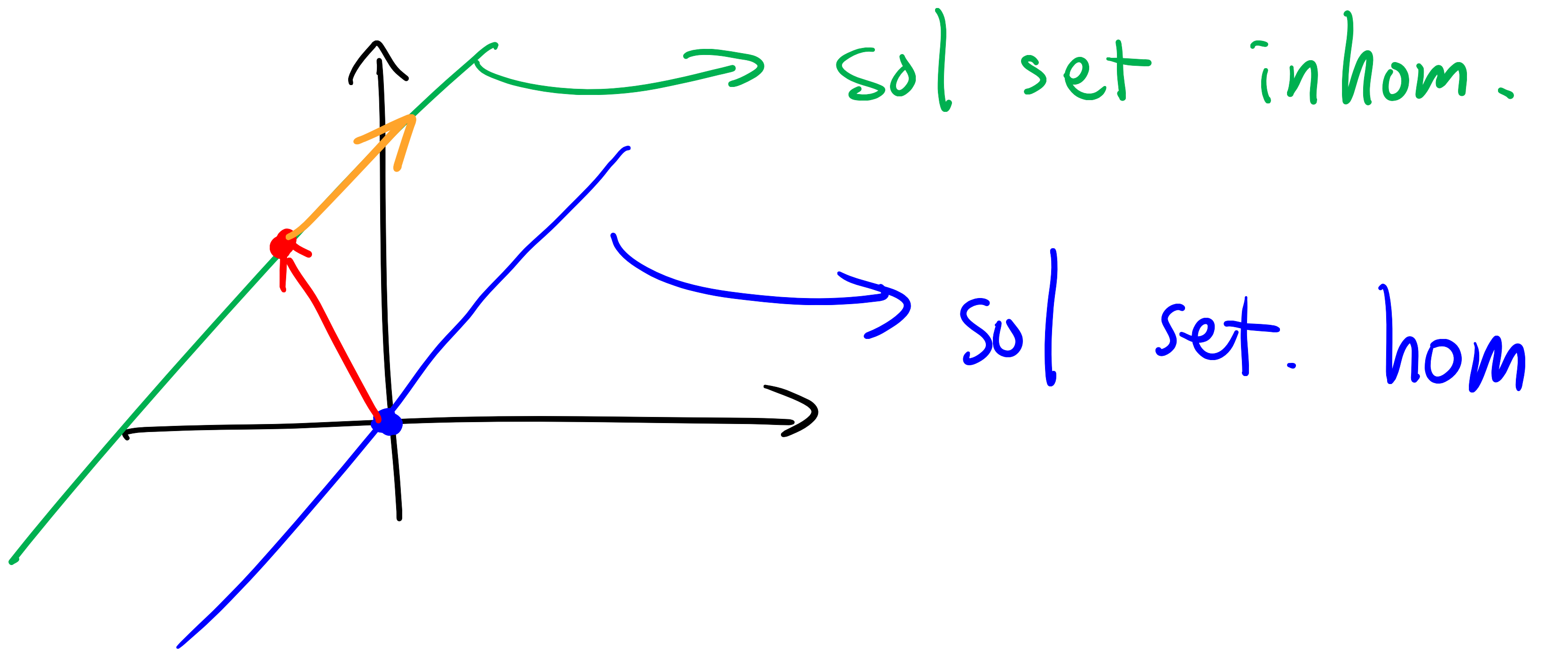
(1) $\vec{0} \in S$

(2) $\vec{x}, \vec{y} \in S$, $\vec{x}+\vec{y} \in S$

(3) $c \in \mathbb{R}$, $\vec{x} \in S$, $c\vec{x} \in S$.

S is a subspace. of $\mathbb{R}^n$

what about $A\vec{x}=\vec{b}$



$\{\vec{v}_1, \cdots, \vec{v}_k\} \subseteq \mathbb{R}^n$

Then $\operatorname{span}\{\vec{v}_1, \cdots, \vec{v}_k\}$ is a subspace of $\mathbb{R}^n$.

Two important examples of subspace

$A \in \mathbb{R}^{m \times n}$

(1) Column space.

col(A) = span of all column vectors of A.

$$A = [\vec{a}_1, \cdots, \vec{a}_n].$$

$$\mathrm{col}(A) = \mathrm{span}\{\vec{a}_1, \cdots, \vec{a}_n\}$$

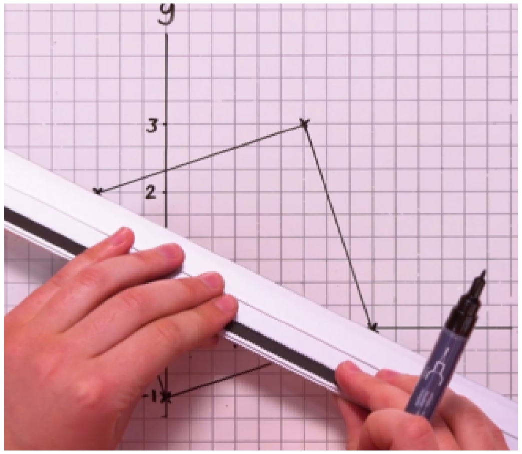
a subspace of $\mathbb{R}^m$

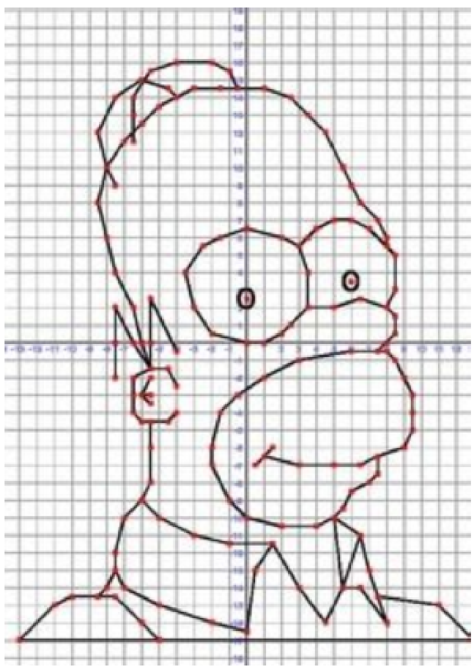
(2) Null space.

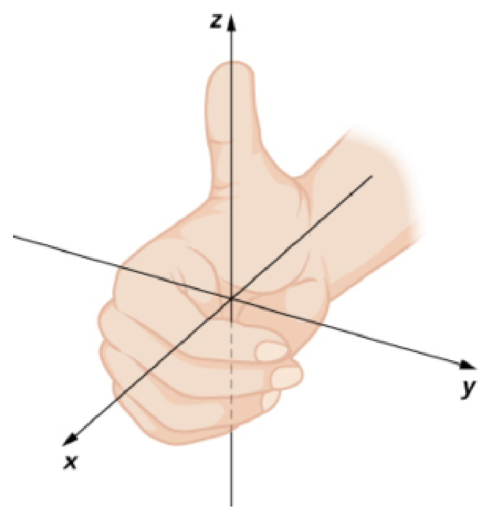
$\mathrm{Null}(A)$ = sol set of $A\vec{x} = \vec{0}$.

a subspace of $\mathbb{R}^n$

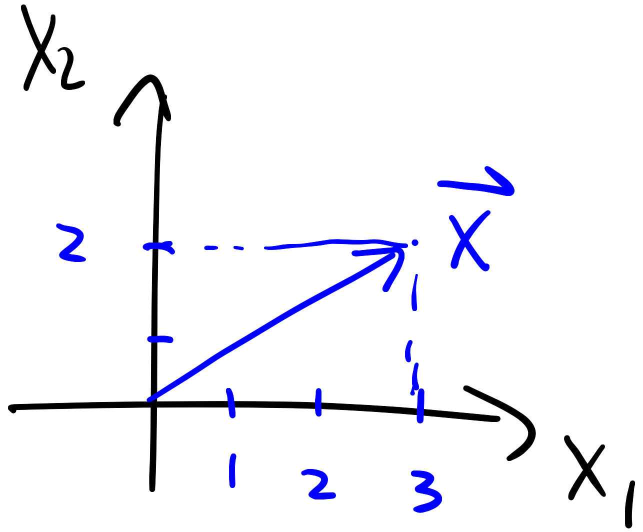
What is coordinate







Ex.



$$\vec{b}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \vec{b}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\vec{x} \in \mathbb{R}^2.$$

$$\vec{x} = x_1 \vec{b}_1 + x_2 \vec{b}_2 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$[\vec{x}]_B = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

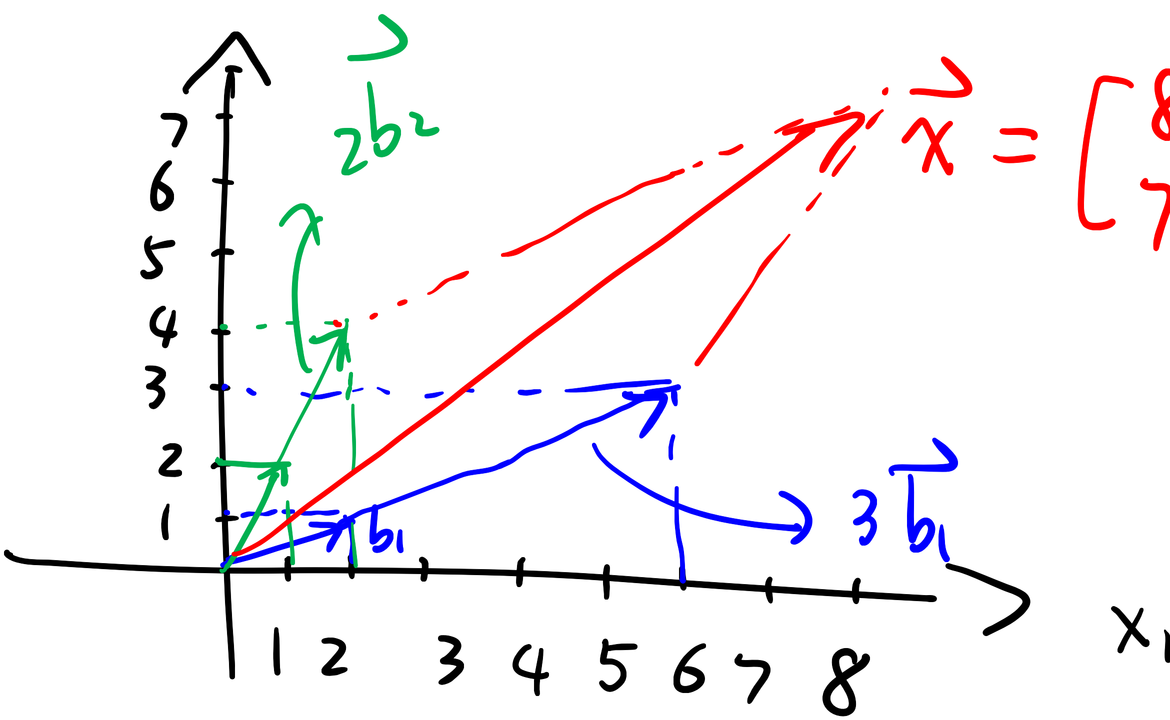
$B = \{\vec{b}_1, \vec{b}_2\}$ basis

"standard basis"

Ex. $\vec{b}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\vec{b}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\vec{x} = 3\vec{b}_1 + 2\vec{b}_2$. $\mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$

$[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$



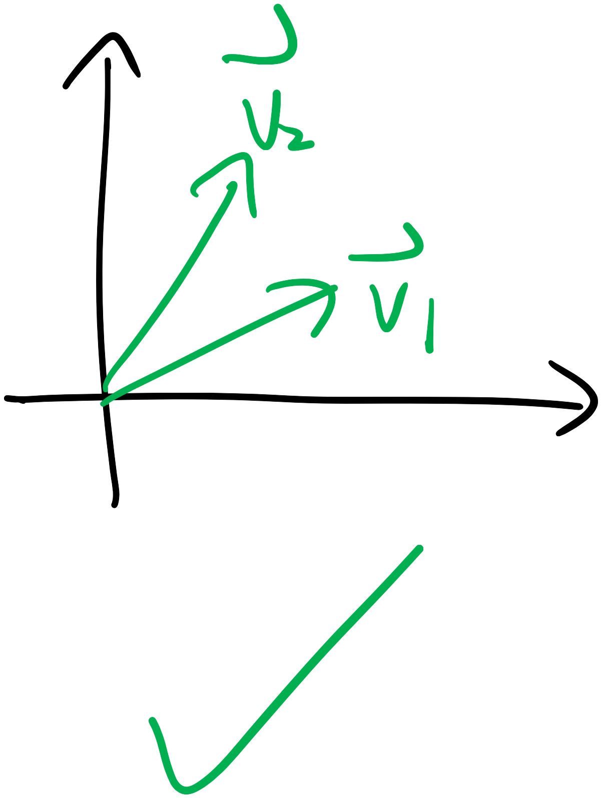
$\vec{x} = \begin{bmatrix} 8 \\ 7 \end{bmatrix}$.

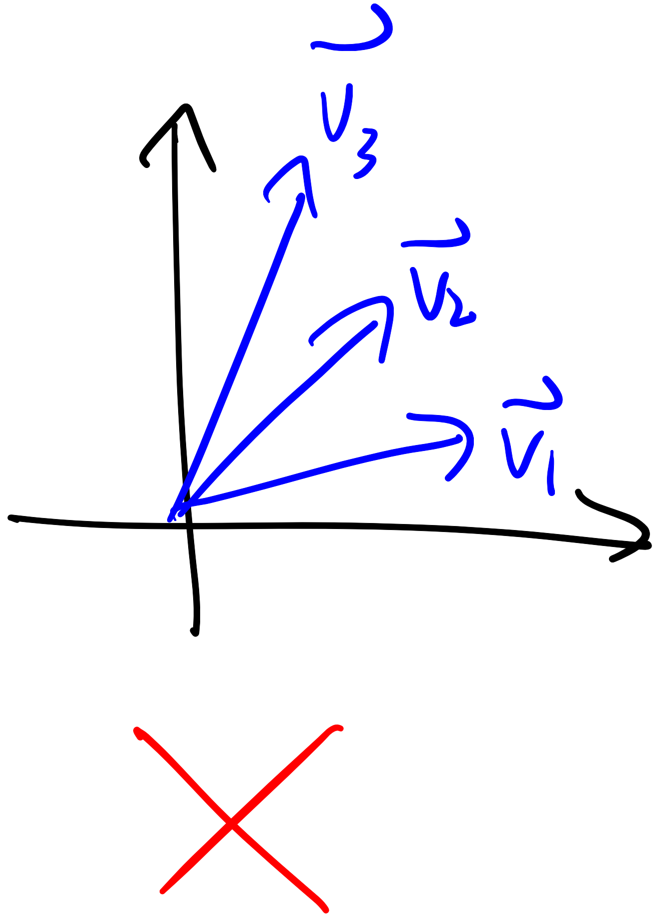
Def A basis for a subspace H of $\mathbb{R}^n$ is an ordered set of vectors $\{\vec{v}_1, \dots, \vec{v}_k\}$ s.t.

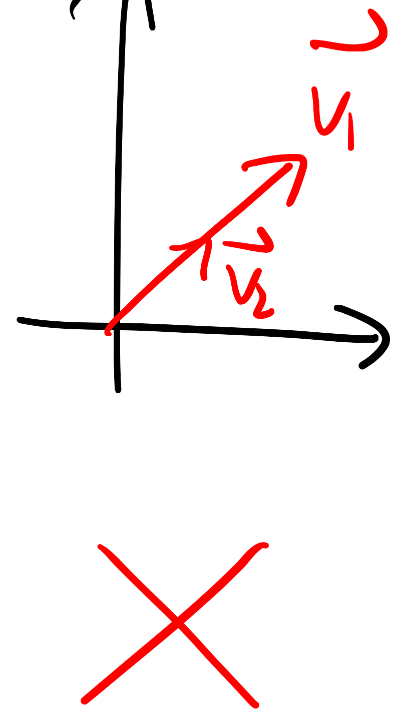
(1) $\{\vec{v}_1, \dots, \vec{v}_k\}$ spans H (big enough)

(2) $\{\vec{v}_1, \dots, \vec{v}_k\}$ lin. indep. (small enough)

Ex. $H = \mathbb{R}^2$







Coordinate.

$\mathcal{B} = \{\vec{b_1}, \cdots, \vec{b_p}\}$ basis of subspace H. Given $\vec{x} \in H$

$$\vec{x} = c_1 \vec{b_1} + \cdots + c_p \vec{b_p}$$

$$[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ \vdots \\ c_p \end{bmatrix}$$ is called

coordinates of $\vec{x}$ relative to B.

<u>Def</u> H is a subspace of $\mathbb{R}^n$.

the dimension of H

$\dim(H)$ = size of ANY basis of H.